

2018-2019 Curriculum Guide

October 15 -October 31

<u>Eureka</u>

Module 2: Addition and Subtraction of Length Units



ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

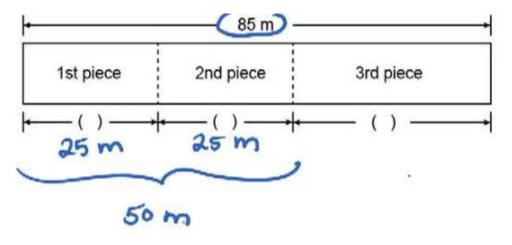
OFFICE OF MATHEMATICS

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Module 2 Performance Overview

- Topic A opens with students exploring concepts related to the centimeter ruler. In the first lesson, they are guided to connect measurement with physical units as they find the total number of length units by laying multiple copies of centimeter cubes (physical units) end to end along various objects. Through this, students discover that to get an accurate measurement, there must be no gaps or overlaps between consecutive length units. Next, students measure by iterating with one physical unit, using the mark and advance technique, also known as mark and move forward. Students then repeat the process by laying both multiple copies and a single cube along a centimeter ruler. This helps students create a mental benchmark for the centimeter. It also helps them realize that the distance between 0 and 1 on the ruler indicates the amount of space already covered. Hence 0, not 1, marks the beginning of the total length. Students use this understanding to create their own centimeter rulers using a centimeter cube and the mark and advance technique. Topic A ends with students using their unit rulers to measure lengths, thereby connecting measurement with a ruler. Students build skill in measuring using centimeter rulers and meter sticks
- In Topic B, students learn to see that a length unit is not a cube, or a portion of a ruler (which has width) but is a segment of a line. By measuring a variety of objects, students build a bank of known measurements or benchmark lengths, such as a doorknob being a meter from the floor, or the width of a finger being a centimeter. Students learn to estimate length using knowledge of previously measured objects and benchmarks. This enables students to internalize the mental rulers of a centimeter or meter, empowering them to mentally iterate units relevant to measuring a given length. The knowledge and experience signal that students are determining which tool is appropriate to make certain measurements
- In Topic C, students measure and compare to determine how much longer one object is than another. They also measure objects twice using different length units, both standard and non-standard, thereby developing their understanding of how the total measurement relates to the size of the length unit. Repeated experience and explicit comparisons help students recognize that the smaller the length unit, the larger the number of units, and the larger the length unit, the smaller the number of units.



Module 2: Addition and Subtraction of Length Units Pacing:

October 15- October 31

10 Davs

		10 Days
Topic	Lesson	Student Lesson Objective/ Supportive Videos
	Lesson	Connect measurement with physical units by using mul-
Topic A:	1	tiple copies of the same physical unit to measure
Understand		https://www.youtube.com/watch?v
Concepts	Lesson	Use iteration with one physical unit to measure.
about the	2&3	Apply concepts to create unit rulers and measure lengths
Ruler		using unit rulers
		https://www.youtube.com/watch?v
		https://www.youtube.com/watch?v
	Lesson	Measure various objects using centimeter rulers and me-
Topic B:	4 & 5	ter sticks
_		Develop estimation strategies by applying prior
Measure and Estimate		knowledge of length and using mental benchmarks
Length Using		https://www.youtube.com/watch?v
Different		https://www.youtube.com/watch?v
Measurement		arcepsity www.youtensorously.watesirv
Tools		
	Lesson	Measure and compare lengths using centimeters and
Topic C:	6	meters
Measure and		
Compare		https://www.youtube.com/watch?v
Lengths Using	Lesson	Measure and compare lengths using standard metric
Different	7	length units and non-standard length units; relate
Length Units	_	measurement to unit size
		https://www.youtube.com/watch?v
	T	Solve addition and subtraction word problems using the
	Lesson 8	ruler as a number line.
		https://www.youtube.com/watch?v
Topic D:	Lesson	Measure lengths of string using measurement tools and
Relate	9	use tape diagrams to represent and compare the lengths.
Addition and		https://www.woutube.com/watch?w
Subtraction to Length	Т.	https://www.youtube.com/watch?v Apply conceptual understanding of measurement by
Longui	Lesson 10	solving two-step word problems
	10	S t p p
		https://www.youtube.com/watch?v
	E	End-of- Module Assessment
		October 30-31,2018

NJSLS Standards:

Module 2: Addition and Subtraction of Length Units

2.MD.1

Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Second Graders build upon their non-standard measurement experiences in First Grade by measuring in standard units for the first time. Using both customary (inches and feet) and metric (centimeters and meters) units, Second Graders select an attribute to be measured (e.g., length of classroom), choose an appropriate unit of measurement (e.g., yardstick), and determine the number of units (e.g., yards). As teachers provide rich tasks that ask students to perform real measurements, these foundational understandings of measurement are developed:

- Understand that larger units (e.g., yard) can be subdivided into equivalent units (e.g., inches) (partition).
- Understand that the same object or many objects of the same size such as paper clips can be repeatedly used to determine the length of an object (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principal). Thus, the smaller the unit, the more units it will take to measure the selected attribute.

When Second Grade students are provided with opportunities to create and use a variety of rulers, they can connect their understanding of non-standard units from First Grade to standard units in second grade. For example:

By helping students progress from a "ruler" that is blocked off into colored units (no numbers)	
to a "ruler" that has numbers along with the colored units	1 2 3 4 5 6 7 8
to a "ruler" that has inches (centimeters) with and without numbers, students develop the understanding that the numbers on a ruler do not count the individual marks but indicate the spaces (distance) between the marks. This is a critical understand students	
need when using such tools as rulers, yardsticks, meter sticks, and measuring tapes.	

By the end of Second Grade, students will have also learned specific measurements as it relates to feet, yards and meters:

- There are 12 inches in a foot.
- There are 3 feet in a yard.

There are 100 centimeters in a meter

2.MD.2

Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen

Second Grade students measure an object using two units of different lengths. This experience helps students realize that the unit used is as important as the attribute being measured. This is a difficult concept for young children and will require numerous experiences for students to predict, measure, and discuss outcomes.

<u>Example</u>: A student measured the length of a desk in both feet and inches. She found that the desk was 3 feet long. She also found out that it was 36 inches long.

Teacher: Why do you think you have two different measurements for the same desk?

Student: It only took 3 feet because the feet are so big. It took 36 inches because an inch is a whole lot smaller than a foot.

2.MD.3

Estimate lengths using units of inches, feet, centimeters, and meters

Second Grade students estimate the lengths of objects using inches, feet, centimeters, and meters prior to measuring. Estimation helps the students focus on the attribute being measured and the measuring process. As students estimate, the student has to consider the size of the unit-helping them to become more familiar with the unit size. In addition, estimation also creates a problem to be solved rather than a task to be completed. Once a student has made an estimate, the student then measures the object and reflects on the accuracy of the estimate made and considers this information for the next measurement.

Example:

Teacher: How many inches do you think this string is if you measured it with a ruler? **Student**: An inch is pretty small. I'm thinking it will be somewhere between 8 and 9 inches.

Teacher: Measure it and see.

Student: It is 9 inches. I thought that it would be somewhere around there.

2.MD.4

Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit

Second Grade students determine the difference in length between two objects by using the same tool and unit to measure both objects. Students choose two objects to measure, identify an appropriate tool and unit, measure both objects, and then determine the differences in lengths.

Example:

Teacher: Choose two pieces of string to measure. How many inches do you think each string is?

Student: I think String A is about 8 inches long. I think string B is only about 4 inches long. It's really short. **Teacher**: Measure to see how long each string is. *Student measures*. What did you notice?

Student: String A is definitely the longest one. It is 10 inches long. String B was only 5 inches long. I was close! **Teacher**: How many more inches does your short string need to be so that it is the same length as your long string?

Student: Hmmm. String B is 5 inches. It would need 5 more inches to be 10 inches. 5 and 5 is 10

2.MD.5

Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

Second Grade students apply the concept of length to solve addition and subtraction word problems with numbers within 100. Students should use the same unit of measurement in these problems. Equations may vary depending on students' interpretation of the task. Notice in the examples below that these equations are similar to those problem types in Table 1 at the end of this document.

<u>Example</u>: In P.E. class Kate jumped 14 inches. Mary jumped 23 inches. How much farther did Mary jump than Kate? Write an equation and then solve the problem.

Student A

My equation is $14 + \underline{\hspace{0.2cm}} = 23$ since I thought, "14 and what makes 23?". I used Unifix cubes. I made a train of 14. Then I made a train of 23. When I put them side by side, I saw that Kate would need 9 more cubes to be the same as Mary. So, Mary jumped 9 more inches than Kate. 14 + 9 = 23.

 	 	 	 		 _

Student B

My equation is 23 - 14 =__ since I thought about what the difference was between Kate and Mary. I broke up 14 into 10 and 4. I know that 23 minus 10 is 13. Then, I broke up the 4 into 3 and 1. 13 minus 3 is 10. Then, I took one more away. That left me with 9. So, Mary jumped 9 more inches than Kate. That seems to make sense since 23 is almost 10 more than 14. 23 - 14 = 9.

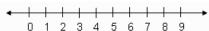
$$23 - 10 = 13$$

 $13 - 3 = 10$
 $10 - 1 = 9$

2.MD.6

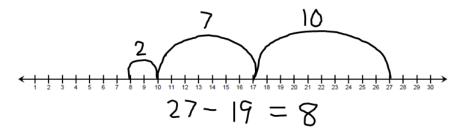
Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Building upon their experiences with open number lines, Second Grade students create number lines with evenly spaced points corresponding to the numbers to solve addition and subtraction problems to 100. They recognize the similarities between a number line and a ruler.



Example: There were 27 students on the bus. 19 got off the bus. How many students are on the bus?

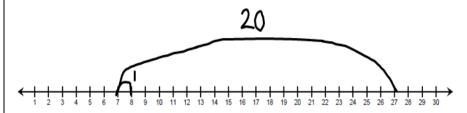
Student A: I used a number line. I started at 27. I broke up 19 into 10 and 9. That way, I could take a jump of 10. I landed on 17. Then I broke the 9 up into 7 and 2. I took a jump of 7. That got me to 10. Then I took a jump of 2. That's 8. So, there are 8 students now on the bus.



Student B: I used a number line. I saw that 19 is really close to 20. Since 20 is a lot easier to work with, I took a jump of 20. But, that was one too many. So, I took a jump of 1 to make up for the extra. I landed on 8. So, there are 8 students on the bus.

$$27 - 20 = 7$$

 $7 + 1 = 8$



M: Major Content

S: Supporting Content

A : Additional Content

Common addition and subtraction. 1

	RESULT UNKNOWN	CHANGEUNKNOWN	START UNKNOWN
	Two bunnies sat on the grass.	Two bunnies were sitting on	Some bunnies were sitting on
	Three more bunnies hopped	the grass. Some more bunnies	the grass. Three more bunnies
ADDTO	there. How many bunnies are	hopped there. Then there were	hopped there. Then there wer
ADD TO	on the grass now? 2+3=?	five bunnies. How many	five bunnies. How many
		bunnies hopped over to the	bunnies were on the grass
		first two? 2 + ? = 5	before? ?+3=5
	Five apples were on the table. I	Five apples were on the table. I	Some apples were on the table
	ate two apples. How many	ate some apples. Then there	Late two apples. Then there
TAKE FROM	apples are on the table now?5-	were three apples. How many	were three apples. How many
	2 = ?	apples did I eat?5 - ? = 3	apples were on the table
			before? ?-2=3
	TOTAL UNKNOWN	ADDEND UNKNOWN	BOTH ADDENDS
			UNKNOWN ²
	Three red apples and two green	Five apples are on the table.	Grandma has five flowers. Ho
PUT TOGETHER /	apples are on the table. How	Three are red and the rest are	many can she put in the red
TAKE APART 3	many apples are on the table? 3	green. How many apples are	vase and how many in her blue
IAKE APART	+2=?	green? 3+?=5,5-3=?	vase? 5 = 0 + 5, 5 + 0 5 = 1 +4
			= 4+1, 5 = 2+3, 5 = 3+2
COMPARE	DIFFERENCE UKNOWN	BIGGER UNKNOWN	SMALLER UNKNOWN
	("How many more?"	(Version with "more"): Julie has	(Version with "more"):Julie ha
	version):Lucy has two apples.	three more apples than	three more apples than Lucy.
	Julie has five apples. How many	Lucy. Lucy has two apples. How	Julie has five apples. How mar
	more apples does Julie have	many apples does Julie have?	apples does Lucy have?(Version
	than Lucy?("How many fewer?"	(Version with "fewer"): Lucy has	with "fewer"): Lucy has 3 fewer
	version): Lucy has two apples.	3 fewer apples than Julie. Lucy	apples than Julie. Julie has five
	Julie has five apples. How many	has two apples. How many	apples. How many apples doe
	fewer apples does Lucy have	apples does Julie have? 2 + 3 =	Lucy have? 5 - 3 = ?, ? + 3 = 5

Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/

² These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the – sign does not always mean, makes or results in but always does mean is the same number as.

³ Either addend can be unknown, so there are three variations of these problem situations. Both addends Unknown is a productive extension of the basic situation, especially for small numbers less than or equal to 10.

⁴ For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

Module 2 Assessment / Authentic Assessment Recommended Framework					
Assessment CCSS Estimated Format Time					
<u>Eureka Math</u> <u>Module 2: Add/ Subtract Length</u>					
End-of-Module Assessment	2.MD.1-6	1 Block	Individual		

Second Grade Ideal Math Block

Fluency: Whole Group

Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group

Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

50-60 min.

15-20 min.

Concept Development: Individual/partner/whole

Instruction & Strategic Problem Set Questions

Student Debrief: Whole Group

Exit Ticket: Independent

CENTERS/STATIONS:

Pairs / Small Group/ Individual

DIFFERENTIATED activities designed to **RETEACH**, **REMEDIATE**, **ENRICH** student's understanding of concepts.

M: Meet with the

teacher

A:
Application/
Problem Solving

T: Technology H: Hands on Activities

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting: Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

	PARCC Assessment Evidence/Clarification Statements					
ccss	Evidence Statement	Clarification	Math Practices			
2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes	i)Length may be measured in whole units within the same measurement system using metric or U.S. customary. ii) Units are limited to those found in 2.MD.3	MP 5			
2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.	i)Tasks should be limited to whole units within the same measurement system. ii) Units are limited to those found in 2.MD.3 iii) Example: Student measures the length of a table in inches and in feet and notes that the number of feet is less than the number of inches because an inch is smaller than a foot. Therefore, it takes more inch units than foot units to measure the table's length	MP 5			
2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.	i) Rulers are not used to estimate.	MP 5,6			
2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.	i.Length may be measured in whole units within the same measurement system using metric or U.S. customary. ii. Units are limited to those in 2.MD.3.	MP 5,6			

Number Talks Cheat Sheet

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

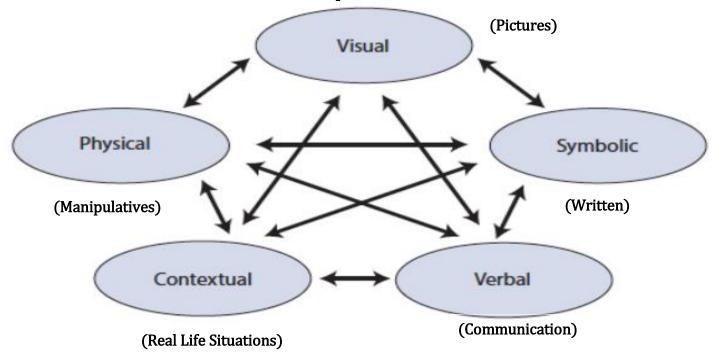
The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?
- What was the first thing your eyes saw, or your brain did?
- What are Number Talks and Why are they

Student Name:	Task:	School:	Teacher:	Date:
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	STUDENT FRIENDLY RUBRIC				
"I CAN"	a start 1	getting there 2	that's it 3	WOW! 4	SCORE
Understand	I need help.	I need some help.	I do not need help.	I can help a class- mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my thinking.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over.... And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing? **WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

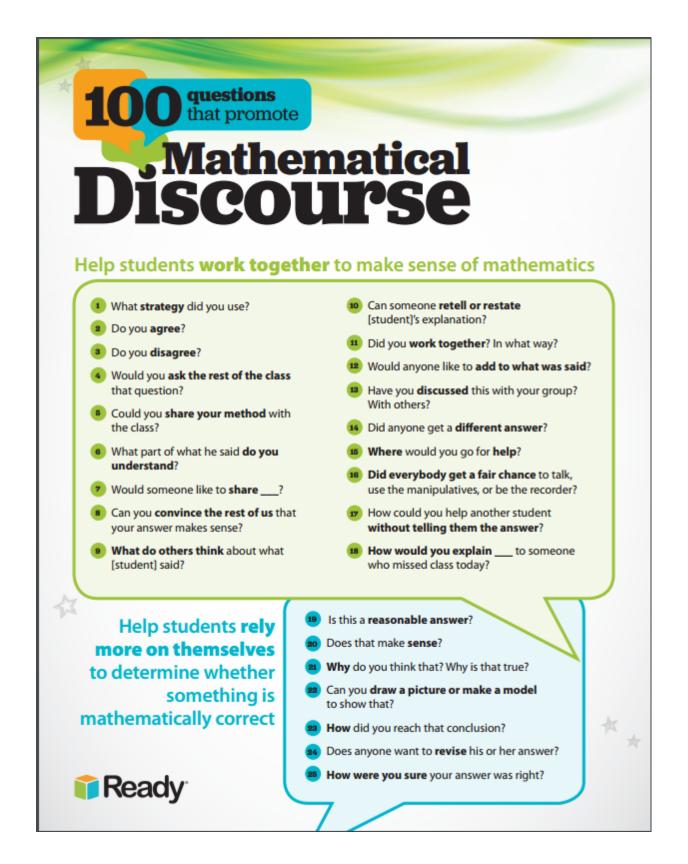
Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



Help students learn to reason mathematically

- How did you begin to think about this problem?
- What is another way you could solve this problem?
- How could you prove ____
- Can you explain how your answer is different from or the same as [student]'s answer?
- Let's break the problem into parts. What would the parts be?
- Can you explain this part more specifically?
- Does that always work?
- Can you think of a case where that wouldn't work?
- 34 How did you organize your information? Your thinking?

Help students with problem comprehension

Help students evaluate their own processes and engage in productive peer interaction

- What do you need to do next?
- 36 What have you accomplished?
- What are your strengths and weaknesses?
- Was your group participation appropriate and helpful?
 - What is this problem about? What can you tell me about it?
 - O Do you need to define or set limits for the problem?
 - How would you interpret that?
 - Could you reword that in simpler terms?
 - 43 Is there something that can be eliminated or that is missing?
 - Could you explain what the problem is asking?
 - What assumptions do you have to make?
 - What do you know about this part?
 - Which words were most important? Why?



100 Questions That Promote Mathematical Discourse 2



- What would happen if ___?
- Do you see a pattern?
- What are some possibilities here?
- Where could you find the information you need?
- How would you check your steps or your answer?
- What did not work?
- How is your solution method the same as or different from [student]'s method?
- Other than retracing your steps, how can you determine if your answers are appropriate?
- 66 How did you organize the information? Do you have a record?
- How could you solve this using tables, lists, pictures, diagrams, etc.?
- What have you tried? What steps did you take?
- 69 How would it look if you used this model or these materials?

- How would you draw a diagram or make a sketch to solve the problem?
- 61 Is there another possible answer? If so, explain.
- Is there another way to solve the problem?
- Is there another model you could use to solve the problem?
- Is there anything you've overlooked?
- How did you think about the problem?
- 66 What was your estimate or prediction?
- How confident are you in your answer?
- What else would you like to know?
- What do you think comes next?
- Is the solution reasonable, considering the context?
- Did you have a system? Explain it.
- Did you have a strategy? Explain it.
- Did you have a design? Explain it.





100 Questions That Promote Mathematical Discourse 3

Help students learn to connect mathematics, its ideas, and its application

- What is the relationship between ____
- Have we ever solved a problem like this before?
- What uses of mathematics did you find in the newspaper last night?
- What is the same?
- What is different?
- Did you use skills or build on concepts that were not necessarily mathematical?
- Which skills or concepts did you use?
- What ideas have we explored before that were useful in solving this problem?

- Is there a pattern?
- Where else would this strategy be useful?
- How does this relate to ?
- Is there a general rule?
- Is there a real-life situation where this could be used?
- How would your method work with other problems?
- What other problem does this seem to lead to?
 - Have you tried making a guess?
 - What else have you tried?
 - Would another method work as well or better?
 - 92 Is there another way to draw, explain, or say that?
 - Give me another related problem. Is there an easier problem?
 - How would you explain what you know right now?

Help students persevere

- What was one thing you learned (or two, or more)?
- Did you notice any patterns? If so, describe them.
- What mathematics topics were used in this investigation?
- What were the mathematical ideas in this problem?
- What is mathematically different about these two situations?
- What are the variables in this problem? What stays constant?

Help students focus on the mathematics from activities

Ready

100 Questions That Promote Mathematical Discourse 4

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the <u>mind</u> with the low-level details required, allowing it to become an automatic response pattern or <u>habit</u>. It is usually the result of <u>learning</u>, <u>repetition</u>, and practice.

K-2 Math Fact Fluency Expectation

K.OA.5 Add and Subtract within 5.

1.0A.6 Add and Subtract within 10.

2.0A.2 Add and Subtract within 20.

Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

1.0A.6 Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 4 = 13 3 1 = 10 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

2.NBT.7 Add and subtract within 1000, using concrete models or drawings and strategies based on:

- o place value,
- o properties of operations, and/or
- o the relationship between addition and subtraction;

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

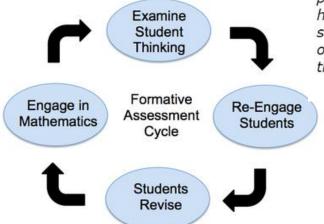
Mathematical Proficiency

To be mathematically proficient, a student must have:

- <u>Conceptual understanding</u>: comprehension of mathematical concepts, operations, and relations;
- <u>Procedural fluency</u>: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- <u>Strategic competence</u>: ability to formulate, represent, and solve mathematical problems;
- <u>Adaptive reasoning</u>: capacity for logical thought, reflection, explanation, and justification;
- <u>Productive disposition</u>: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

- 1. I can solve problems without giving up.
- 2. I can think about numbers in many ways.
- 3. I can explain my thinking and try to understand others.
- 4. I can show my work in many ways.
- 5. I can use math tools and tell why I choose them.
- 6. I can work carefully and check my work.
- 7. I can use what I know to solve new problems.
- 8. I can discover and use short cuts.

ards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in the
n make sense of the meaning of the task and find an entry point or a way to start the task. Second Grade students continues to use concrete manipulatives and pictorial representations as well as mental mathematics. Second Gractinue to solve the task. Lastly, mathematically proficient students complete a task by asking themselves the questio
ationships while solving tasks. This involves two processes- decontextualizing and contextualizing. In Second Grade of more children. How many students are in the cafeteria? "Second Grade students translate that situation into an enterent of the context of the task to determine that they need to subtract 19 since 19 children leave. The students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave. The students can refer to the context of the task to determine that they need to subtract 19 since 19 children leave.
eviously established solutions to construct viable arguments about mathematics. During discussions about problem gies, and after working on the task, can discuss and critique each others' reasoning and strategies, citing similarities
ations with a number sentence or an equation, and check to make sure that their equation accurately matches the
idents are able to create an appropriate problem situation from an equation. For example, students are expected to
nachine?"

propriately. These tools may include snap cubes, place value (base ten) blocks, hundreds number boards, number anipulatives, which support conceptual understanding and higher-order thinking skills. During classroom instruction of the hallway, students can explain why a yardstick is more appropriate to use than a ruler.
on, calculations, and measurements. In all mathematical tasks, students in Second Grade communicate clearly, usi an object, care is taken to line up the tool correctly in order to get an accurate measurement. During tasks involving
ructures in the number system and other areas of mathematics. For example, students notice number patterns with ts work with the idea that 10 ones equals a ten, and 10 tens equals 1 hundred. In addition, Second Grade students uch more do I need to add to 33 to get to 50?"
blem structures when solving mathematical tasks. For example, after solving two digit addition problems by decomp strategies to be more efficient in computations, including doubles strategies and making a ten. Lastly, while solving

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions					
Practice	Description/ Questions				
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task?				
	How do you respond to the work that students are likely to produce?				
	Which strategies from student work will be most useful in addressing the mathematical goals?				
2. Monitoring	Paying attention to what and how students are thinking during the lesson.				
	Students working in pairs or groups				
	Listening to and making note of what students are discussing and the strategies they are using				
	Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)				
3. Selecting	This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.				
4. Sequencing	What order will the solutions be shared with the class?				
5. Connecting	Asking the questions that will make the mathematics explicit and understandable.				
	Focus must be on mathematical meaning and relationships; making links between				
	mathematical ideas and representations.				

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.



In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.

In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

ath Workstation:		 	Time:
JSLS.:			
	this task, I will be able to:		
•			
sk(s):			
<u> </u>			
•			
•			
•it Ticket:			

MATH WORKSTATION SCHEDULE

Week	of.
VVCCK	UI.

DAY	Technology	Problem Solving Lab	Fluency	Math	Small Group Instruc-
DAI	3.	1 Toblem Solving Lab			- 1
	Lab		Lab	Journal	tion
Mon.					
	Group	Group	Group	Group	BASED
Tues.					ON CURRENT
	Group	Group	Group	Group	OBSERVATIONAL DATA
Wed.					DATA
	Group	Group	Group	Group	
Thurs.					
	Group	Group	Group	Group	
Fri.	_		•		
	Group	Group	Group	Group	

INSTRUCTIONAL GROUPING

	INSTRUCTION ILL GROOTING						
	GROUP A		GROUP B				
1		1					
2		2					
3		3					
4		4					
5		5					
6		6					
	GROUP C		GROUP D				
1		1					
2		2					
3		3					
4		4					
5		5					
6		6					

Second Grade PLD Rubric

Go	t It	Second Grade PLD Rubric Not There Yet			
Evidence shows that the student essentially has the target con-		Student shows evidence of a mai		oncents or procedure, or a fail-	
cept or big math idea.	t essentially has the target con	Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.			
PLD Level 4: 89%		PLD Level 3: 79%	PLD Level 2: 69%	PLD Level 1: 59%	
Distinguished command	Strong Command	Moderate Command	Partial Command	Little Command	
Student work shows distin-	Student work shows strong	Student work shows moderate	Student work shows partial	Student work shows little un-	
guished levels of understand-	levels of understanding of the	levels of understanding of the	understanding of the mathe-	derstanding of the mathemat-	
ing of the mathematics.	mathematics.	mathematics.	matics.	ics.	
Student constructs and com-	Student constructs and com-	Student constructs and com-	Student constructs and com-	Student attempts to constructs	
municates a complete re-	municates a complete re-	municates a complete response	municates an incomplete re-	and communicates a response	
sponse based on explana-	sponse based on explana-	based on explana-	sponse based on student's at-	using the:	
tions/reasoning using the:	tions/reasoning using the:	tions/reasoning using the:	tempts of explanations/ rea-	• Tools:	
• Tools:	• Tools:	• Tools:	soning using the:	 Manipulatives 	
 Manipulatives 	 Manipulatives 	 Manipulatives 	• Tools:	o Five Frame	
o Five Frame	o Five Frame	o Five Frame	 Manipulatives 	o Ten Frame	
o Ten Frame	o Ten Frame	o Ten Frame	o Five Frame	o Number Line	
o Number Line	o Number Line	o Number Line	o Ten Frame	o Part-Part-Whole	
o Part-Part-Whole	o Part-Part-Whole	o Part-Part-Whole	o Number Line	Model	
Model	Model	Model	o Part-Part-Whole	• Strategies:	
Strategies:	Strategies:	Strategies:	Model	o Drawings	
o Drawings	o Drawings	o Drawings	Strategies:	o Counting All	
o Counting All	o Counting All	o Counting All	o Drawings	o Count On/Back	
o Count On/Back	o Count On/Back	o Count On/Back	o Counting All	Skip Counting Malaine Tens	
Skip Counting Making Ton	Skip Counting Making Top	Skip Counting Making Top	o Count On/Back	Making Ten Decomposing	
Making Ten Decomposing	Making TenDecomposing	o Making Ten	Skip CountingMaking Ten	DecomposingNumber	
DecomposingNumber	 Decomposing Number 	DecomposingNumber	Making TenDecomposing	Precise use of math vo-	
Precise use of math vo-	Precise use of math vo-	Precise use of math vo-	Number	cabulary	
cabulary	cabulary	cabulary	Precise use of math vo-	Cabulary	
Response includes an efficient			cabulary	Response includes limited evi-	
and logical progression of Response includes a logical		Response includes a logical but	Cabalar y	dence of the progression of	
mathematical reasoning and	progression of mathematical	incomplete progression of	Response includes an incom-	mathematical reasoning and	
understanding. reasoning and understanding.		mathematical reasoning and	plete or illogical progression of	understanding.	
S S		understanding.	mathematical reasoning and	3	
		Contains minor errors.	understanding.		
5 points	4 points	3 points	2 points	1 point	

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form	School:			
Assessment:		NJSLS:		
GROUPS (STUDENT INITIALS) MASTERED (86% - 100%) (PLD 4/5):	SUPPORT PLAN		PROGRESS	
DEVELOPING (67% - 85%) (PLD 3):				
INSECURE (51%-65%) (PLD 2):				
BEGINNING (0%-50%) (PLD 1):				

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

K-2 GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews¹.
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended*.
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)².
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

GRADES K-2

Student Portfolio Review

Provide students the opportunity to review and evaluate their portfolio at various points throughout the year; celebrating their progress and possibly setting goals for future growth. During this process, students should retain ALL of their current artifacts in their Mathematics Portfoli

Resources

Great Minds

https://greatminds.org/

Embarc

https://embarc.online/

Number Book Assessment Link: http://investigations.terc.edu/

Model Curriculum- http://www.nj.gov/education/modelcurriculum/

Georgia Department of Education: Games to be played at centers with a partner or small group. http://ccgpsmathematicsk-5.wikispaces.com/Kindergarten

Engage NY: *For additional resources to be used during centers or homework.

https://www.engageny.org/sites/default/files/resource/attachments/math-gk-m1-full-module.pdf

Add/ Subtract Situation Types: Darker Shading indicates Kindergarten expectations https://achievethecore.org/content/upload/Add%20Subtract%20Situation%20Types.pdf

Math in Focus PD Videos: https://www-

<u>k6.thinkcentral.com/content/hsp/math/hspmath/common/mif_pd_vid/9780547760346_te/index.</u> html

Number Talk/Strings: https://elementarynumbertalks.wordpress.com/second-grade-number-talks/

Suggested Literature

Fish Eyes by, Lois Ehlert

Ten Little Puppies by, Elena Vazquez

Zin! Zin! Zin! A Violin! by, Lloyd Moss

My Granny Went to the Market by, Stella Blackstone and Christopher Corr

Anno's Couting Book by, Mitsumasa Anno

Chicka, Chicka, 1,2,3 by, Bill Martin Jr.; Michael Sampson; Lois Ehlert

How Dinosaurs Count to 10 by Jane Yolen and Mark Teague

10 Little Rubber Ducks by Eric Carle

Ten Black Dots by Donald Crews

Mouse Count by Ellen Stoll Walsh

Count! by Denise Fleming

21st Century Career Ready Practices

- CRP1. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.
- CRP7. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP9. Model integrity, ethical leadership and effective management.
- CRP10. Plan education and career paths aligned to personal goals.
- CRP11. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" Great Minds. 2018 < https://greatminds.org/account/products>